6-1 Solving Systems by Graphing

Part 1: Determine if an ordered pair is a solution to a system

ex: \((1, -4)\)

\[
\begin{align*}
\begin{cases}
y = -4x \\
y = 2x - 2
\end{cases}
\end{align*}
\]

HINT: substitute 1 in for \(x\) and -4 in for \(y\) in each equation in the system.

\[
\begin{align*}
y &= -4x \\
-4 &= -4(1) \\
-4 &= -4 \quad \sqrt{\text{true}}
\end{align*}
\]

\[
\begin{align*}
y &= 2x - 2 \\
-4 &= 2(1) - 2 \\
-4 &= 0 \quad \sqrt{\text{true}}
\end{align*}
\]

Since the ordered pair only works in one of the equations, it is not a solution to the system.

Exercises:

1. \((1, 3)\)

\[
\begin{align*}
\begin{cases}
2x + y &= 5 \\
-2x + y &= 1
\end{cases}
\end{align*}
\]

2. \((2, -1)\)

\[
\begin{align*}
\begin{cases}
x - 2y &= 4 \\
3x + y &= 6
\end{cases}
\end{align*}
\]

3. \((3, -2)\)

\[
\begin{align*}
\begin{cases}
y &= \frac{1}{3}x - 3 \\
2x + y &= 4
\end{cases}
\end{align*}
\]

Part 2: Solving a system by graphing

ex: \[
\begin{align*}
\begin{cases}
y &= x - 3 \\
y &= -x - 1
\end{cases}
\end{align*}
\]

HINT: re-write the equations in slope-intercept form before graphing if necessary.

Check:

1. \(y = x - 3\)

   \[-2 = 1 - 3\]

   \[-2 = -2 \checkmark\]

2. \(y = -x - 1\)

   \[-2 = -1 - 1\]

   \[-2 = -2 \checkmark\]

Solution \((1, -2)\)
Exercises:

1. \[
\begin{align*}
2x + y &= -1 \\
-x + y &= 5
\end{align*}
\]

2. \[
\begin{align*}
2y &= 2x - 4 \\
4y + \frac{1}{2}x &= -8
\end{align*}
\]

Part 3: Problem Solving

ex:
Mendham Video charges $10 for a membership and $3 per movie rental. Chester Video charges $15 for a membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video stores? What is that cost?

a) **Understand the Problem**
The answer will be the number of videos rented for which the total cost is the same for both video stores. **List important information:**
Mendham Video $10 membership $3 per movie
Chester Video $15 membership $2 per movie

b) **Make a Plan**
Write a system of equations, one equation to represent the price at each video store. Let \(x\) be the number of videos rented and \(y\) be the total cost.
Mendham Video \(y = 3x + 10\)
Chester Video \(y = 2x + 15\)

c) **Solve by Graphing**
The solution to the system is where the two lines intersect on the graph. They appear to cross at \((5, 25)\), so the cost at
both video stores will be the same for **5 video rentals** and that cost will be **$25**.

\[
\begin{align*}
\text{Mendham Video:} & \quad 25 = 3 \times 5 + 10 \\
& \quad 25 = 15 + 10 \\
& \quad 25 = 25 \checkmark \\
\text{Chester Video:} & \quad 25 = 2 \times 5 + 15 \\
& \quad 25 = 10 + 15 \\
& \quad 25 = 25 \checkmark
\end{align*}
\]

**Exercises:**

1. Mr. Marlone is putting money in two savings accounts. Account A started with $200 and Account B started with $300. Mr. Marlone deposits $15 in Account A and $10 in Account B each month. In how many months will the accounts have the same balance? What will the balance be?
6-2 Solving Systems by Substitution

Part 1: Solving a System using Substitution

\[
\begin{align*}
\text{ex:} & \quad \begin{cases}
    x + y = 9 \\
    y = 2x - 3
\end{cases} \\
\text{HINT:} & \quad \text{Follow the five steps to solving a system using substitution.}
\end{align*}
\]

Step 1: Make sure one or both of the equations is solved in terms of \( x \) or \( y \). The second equation is solved in terms of \( y \).

Step 2: Substitute the resulting expression into the other equation.

\[
\begin{align*}
\begin{cases}
    x + y = 9 \\
    y = 2x - 3
\end{cases}
\end{align*}
\]

Now you will have one equation with one variable to solve:

\[
x + (2x - 3) = 9
\]

Step 3: Solve for the variable.

\[
3x - 3 = 9
\]

\[
3x = 12
\]

\[
x = 4
\]

Step 4: Substitute the value of the variable you just solved for into one of the original equations to solve for the other variable.

\[
y = 2(4) - 3
\]

\[
y = 8 - 3
\]

\[
y = 5
\]

Step 5: Write the values of the variables you solved for in steps 3 and 4 as an ordered pair \((x, y)\) and check.

\((4, 5)\)

**Exercises:**

1. \[
\begin{cases}
    y = x + 3 \\
    y = 2x + 5
\end{cases}
\]

2. \[
\begin{cases}
    x = 2y - 4 \\
    x + 8y = 16
\end{cases}
\]

3. \[
\begin{cases}
    2x + y = -4 \\
    x + y = -7
\end{cases}
\]
Part 2: Solving a System using Substitution – Using the Distributive Property

ex: \[
\begin{align*}
4y - 5x &= 9 \\
x - 4y &= 11
\end{align*}
\]

**HINT:** Look in both equations for a variable with a coefficient of 1 or -1 when deciding which equation to solve for x or y.

**Step 1:** Solve equation 2 for x by adding 4y to both sides of the equation.

\[
\begin{align*}
x - 4y &= 11 \\
+ 4y + 4y &= 4y + 11
\end{align*}
\]

**Step 2:** Substitute expression into first equation for x.

\[
\begin{align*}
4y - 5x &= 9 \\
x &= 4y + 11
\end{align*}
\]

\[
4y - 5(4y + 11) = 9
\]

**Step 3:** Solve the equation using distributive property…CAUTION…*don’t forget to distribute the sign with the term*

\[
\begin{align*}
4y - 5(4y) - 5(11) &= 9 \\
4y - 20y - 55 &= 9 \\
-16y - 55 &= 9 \\
-16y &= 64 \\
y &= -4
\end{align*}
\]

**Step 4:** Substitute the value of the variable you just solved for into one of the original equations to solve for the other variable.

\[
\begin{align*}
x &= 4(-4) + 11 \\
x &= -16 + 11 \\
x &= -5
\end{align*}
\]

**Step 5:** Write the values of the variables you solved for in steps 3 and 4 as an ordered pair \((x, y)\) and check.

\((-5, -4)\)

**Equation 1**

<table>
<thead>
<tr>
<th>((-4) - 5(-5))</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-16 + 25)</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

**Equation 2**

<table>
<thead>
<tr>
<th>(-5 - 4(-4))</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5 + 16)</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>
Exercises:

1. \[
\begin{align*}
-2x + y &= 8 \\
3x + 2y &= 9
\end{align*}
\]

2. \[
\begin{align*}
y + 6x &= 11 \\
3x + 2y &= -5
\end{align*}
\]

3. \[
\begin{align*}
-x + y &= 4 \\
3x - 2y &= -7
\end{align*}
\]

6-3 Solving Systems by Elimination

Part 1: Solving a System by Elimination – Using Addition or Subtraction

**ex:** \[
\begin{align*}
x - 2y &= -19 \\
5x + 2y &= 1
\end{align*}
\]

**HINT:** Look for the variables that have the same or opposite coefficients to eliminate, such as -2y and 2y.

**Step 1:** Write the system so that like terms are aligned

**Step 2:** Eliminate one of the variables and solve for the other.

\[
\begin{align*}
x - 2y &= -19 \\
5x + 2y &= 1
\end{align*}
\]

\[
\begin{align*}
6x &= 18 \\
x &= -3
\end{align*}
\]

Since the variable we are going to eliminate, \( y \), has opposite signs, add the two equations together to eliminate.

**Step 3:** Substitute the value of the variable into one of the original equations and solve for the other variable.

\[
\begin{align*}
5(-3) + 2y &= 1 \\
-15 + 2y &= 1 \\
+15 &= +15 \\
2y &= 16 \\
y &= 8
\end{align*}
\]

**Step 4:** Write the answers from Steps 2 and 3 as an ordered pair, \((x, y)\), and check \((-3, 8)\)
Part 2: Solving a System by Elimination – Using Multiplication First

ex: \[
\begin{align*}
2x + y &= 3 \\
-x + 3y &= -12
\end{align*}
\]

**HINT:** Look for a variable in one of the equations that has a coefficient of 1 or -1.

**Step 1:** Write the system so that like terms are aligned

**Step 2:** Multiply one, or both, of the equations by a factor so that the coefficients are the same, or opposites. Eliminate one of the variables and solve for the other.

\[
\begin{align*}
2x + y &= 3 \\
-x + 3y &= -12
\end{align*}
\]

\[
\begin{align*}
(2x + y &= 3) \times 2 \\
-2x + 6y &= -24
\end{align*}
\]

\[
\begin{align*}
2x + y &= 3 \\
-2x + 6y &= -24
\end{align*}
\]

\[
7y = -21
\]

\[
y = -3
\]

**Step 3:** Substitute the value of the variable into one of the original equations and solve for the other variable.

\[
2x + (-3) = 3
\]

\[
\begin{align*}
+3 & \\
2x &= 6
\end{align*}
\]

\[
x = 3
\]

**Step 4:** Write the answers from Steps 2 and 3 as an ordered pair, \((x, y)\), and check

\[
\begin{align*}
2x + y &= 3 \\
-x + 3y &= -12
\end{align*}
\]

\[
\begin{align*}
2(3) + (-3) &= 3 \\
-(3) + 3(-3) &= -12
\end{align*}
\]

\[
\begin{align*}
6 + (-3) &= 3 \\
-(3) - 9 &= -12
\end{align*}
\]

\[
\begin{align*}
3 &= 3 \\
-12 &= -12
\end{align*}
\]
Exercises:

1. \[
\begin{align*}
  y + 3x &= -2 \\
 2y - 3x &= 14
\end{align*}
\]

2. \[
\begin{align*}
  3x + 2y &= 6 \\
  -x + y &= -2
\end{align*}
\]

3. \[
\begin{align*}
  2x + 5y &= 26 \\
 -3x - 4y &= -25
\end{align*}
\]

6-4 Solving Special Systems

**Part 1: Systems with No Solution – Inconsistent**

ex: \[
\begin{align*}
  y &= x - 1 \\
  -x + y &= 2
\end{align*}
\]

Write both equations in Slope-Intercept Form, \( y = mx + b \). Compare the slopes and y-intercepts.

- Equation 1: \( m = 1 \) \( b = -1 \)
- Equation 2: \( m = 1 \) \( b = 2 \)

Since the slopes are the same and the y-intercepts are different, the lines are parallel. They will never have a point of intersection, so there is **No Solution** to this system. It is **Inconsistent**.

**Part 2: Systems with Infinitely Many Solution – Consistent and Dependent**

ex: \[
\begin{align*}
  y &= 2x + 1 \\
 2x - y + 1 &= 0
\end{align*}
\]

Write both equations in Slope-Intercept Form, \( y = mx + b \). Compare the slopes and y-intercepts.

- Equation 1: \( m = 2 \) \( b = 1 \)
- Equation 2: \( m = 2 \) \( b = 1 \)
Since the slopes and the y-intercepts are the same, the lines are also the same. Every point along one line is a point on the other. There are infinitely many points of intersection, so there are **Infinitely Many Solutions** to this system. It is **Consistent and Dependent**.

### Classification of Systems of Linear Equations

<table>
<thead>
<tr>
<th>Classification</th>
<th>Consistent Independent</th>
<th>Consistent Dependent</th>
<th>Inconsistent</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Solutions</td>
<td>Exactly One</td>
<td>Infinitely Many</td>
<td>None</td>
</tr>
<tr>
<td>Description</td>
<td>Different Slopes</td>
<td>Same Slope, Same y-int</td>
<td>Same Slope, Different y-int</td>
</tr>
<tr>
<td>Graph</td>
<td>Two lines, intersecting at one point</td>
<td>One Line</td>
<td>Parallel Lines</td>
</tr>
</tbody>
</table>

### Exercises: Solve and classify each system.

1. \[
\begin{align*}
    y &= 2x - 4 \\
    2x - y &= 4
\end{align*}
\]

2. \[
\begin{align*}
    -y &= 4x + 1 \\
    4x + y &= 2
\end{align*}
\]

3. \[
\begin{align*}
    y &= 2x \\
    x + y &= -6
\end{align*}
\]

### 6-5 Solving Linear Inequalities

**Part 1:** Determine if an ordered pair is a solution to the inequality

ex: \((7,3); \ y \leq x - 1\)

\[
\begin{align*}
    3 &\leq 7 - 1 \\
    3 &\leq 6
\end{align*}
\]

Since the ordered pair makes the inequality a true statement, it is a solution to the inequality.
Part 2: Graphing an inequality in the coordinate plane

ex: \( y < 3x + 4 \)

**Step 1:** Solve the inequality for \( y \) (*slope-intercept form*)

**Step 2:** Graph the line as you would any linear equation, start with the \( y \)-intercept and use slope to find 2 more points. Use a **solid line** for all inclusive signs, \( \leq \) or \( \geq \). Use a **dotted/dashed line** for all non-inclusive signs, \( < \) or \( > \).

**Step 3:** Shade the half-plane above the line for \( y > \) or \( y \geq \). Shade the half-plane below the line for \( y < \) or \( y \leq \).

**Step 4:** Pick a point within the shaded region, a boundary point, to check.

\[ y < 3x + 4 \]
- dotted/dashed line
- shade below

Part 3: Writing an inequality from a graph

ex:
Step 1: Identify the slope and y-intercept from the graph

\[ slope = -2 \quad y - intercept = 3 \]

Step 2: Determine inequality sign from the type of line graphed:
- dotted/dashed is > or < (non-inclusive)
- solid is \( \leq \) or \( \geq \) (inclusive)

the line is solid so it is an inclusive symbol

Step 3: Determine direction of inequality sign by shading:
- shaded below < or \( \leq \)
- shaded above > or \( \geq \)

the shading is below, so the symbol is \( \leq \)

Step 4: Write the inequality using all of this information

\[ y \leq -2x + 3 \]

Exercises:

1. Is the ordered pair a solution? \( (2,3), y \geq 2x + 3 \)

2. Graph: \( y > 3x - 2 \)

3. Write an inequality for the graph below:
6-6 Solving Systems of Linear Inequalities

Part 1: Determine if an ordered pair is a solution to the system

ex: \[
\begin{array}{l}
y < -x + 4 \\
y \leq x + 1
\end{array}
\]

HINT: substitute 2 in for \(x\) and 1 in for \(y\) in each inequality in the system.

\[
\begin{align*}
y &< -x + 4 \\
1 &< -(1)+4 \\
1 &< 3
\end{align*}
\]

\[
\begin{align*}
y &\leq x + 1 \\
1 &\leq 2 + 1 \\
1 &\leq 3
\end{align*}
\]

Since the ordered pair works in both of the inequalities, it is a solution to the system.

Exercises:

1. \[
\begin{array}{l}
y < -3x + 2 \\
y \geq x - 1
\end{array}
\]
2. \[
\begin{array}{l}
y > -x + 1 \\
y > x - 1
\end{array}
\]

Part 2: Solving a system by graphing

ex: \[
\begin{align*}
8x + 4y &\leq 12 \\
y &> \frac{1}{2}x - 2
\end{align*}
\]

\[
\begin{align*}
8x + 4y &\leq 12 \\
y &> \frac{1}{2}x - 2 \\
y &\leq -2x + 3 \\
y &> \frac{1}{2}x - 2
\end{align*}
\]

Step 1: Re-write each inequality in slope-intercept form, if necessary

Step 2: Graph each inequality separately, don’t forget about solid or dotted lines and your shading.

Step 3: Highlight the area where shading from both inequalities overlap. This is your solution.
Exercises:
1. \[
\begin{align*}
  y &\leq x + 1 \\
  y &> -2x + 2
\end{align*}
\]
2. \[
\begin{align*}
  y &> x - 7 \\
  3x + 6y &\leq 12
\end{align*}
\]